

$$\textcircled{3} \text{ Let } |\vec{A}| = A$$

$$\text{Now } \vec{A} \cdot \vec{A} = |\vec{A}|^2 = A^2$$

Differentiating w.r.t.  $t$

$$\frac{d}{dt} (\vec{A} \cdot \vec{A}) = \frac{d}{dt} (A^2)$$

$$\Rightarrow \frac{d\vec{A}}{dt} \cdot \vec{A} + \vec{A} \cdot \frac{d\vec{A}}{dt} = 2A \frac{dA}{dt}$$

$$\left[ \because \frac{d}{dt} (\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt} \right]$$

$$\Rightarrow \vec{A} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{A}}{dt} = 2A \frac{dA}{dt}$$

$$\Rightarrow 2\vec{A} \cdot \frac{d\vec{A}}{dt} = 2A \frac{dA}{dt}$$

$$\Rightarrow \vec{A} \cdot \frac{d\vec{A}}{dt} = A \frac{dA}{dt}$$

Show that if  $\vec{a} = \vec{a}(t)$  is vector function.

$$\frac{d}{dt} \left( \vec{a} \cdot \left( \frac{d\vec{a}}{dt} \times \frac{d^2\vec{a}}{dt^2} \right) \right) = \frac{d^3\vec{a}}{dt^3} \cdot \left[ \vec{a} \times \frac{d\vec{a}}{dt} \right]$$

Sol:  $\frac{d}{dt} \left( \vec{a} \cdot \left( \frac{d\vec{a}}{dt} \times \frac{d^2\vec{a}}{dt^2} \right) \right)$

$$= \frac{d\vec{a}}{dt} \cdot \left( \frac{d\vec{a}}{dt} \times \frac{d^2\vec{a}}{dt^2} \right) + \vec{a} \cdot \frac{d}{dt} \left( \frac{d\vec{a}}{dt} \times \frac{d^2\vec{a}}{dt^2} \right)$$

$$= \left[ \frac{d\vec{a}}{dt} \cdot \frac{d\vec{a}}{dt} \times \frac{d^2\vec{a}}{dt^2} \right] + \vec{a} \cdot \left[ \frac{d^2\vec{a}}{dt^2} \times \frac{d^2\vec{a}}{dt^2} + \frac{d\vec{a}}{dt} \times \frac{d^3\vec{a}}{dt^3} \right]$$

$$= \frac{d^2\vec{a}}{dt^2} \cdot \left( \frac{d\vec{a}}{dt} \times \frac{d\vec{a}}{dt} \right) + \vec{a} \cdot \left[ \frac{d\vec{a}}{dt} \times \frac{d^3\vec{a}}{dt^3} \right]$$

$$= \vec{0} + \frac{d^3\vec{a}}{dt^3} \cdot \left( \vec{a} \times \frac{d\vec{a}}{dt} \right) = \frac{d^3\vec{a}}{dt^3} \cdot \left( \vec{a} \times \frac{d\vec{a}}{dt} \right)$$

∴ vector function.